Third Semester B.E. Degree Examination, June/July 2018
Advanced Mathematics – I

Time: 3 hrs. Max. Ma

Note: ***Answer any FIVE full questions.***

1 a. Find modulus and amplitude of : $z = \frac{(1+i)^2}{1-i}$. (

b. Prove that :

$(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}$. (0

c. If $x = \cos\theta + i\sin\theta$ and $y = \cos\phi + i\sin\phi$, then prove that $\frac{x-y}{x+y} = i\tan\left(\frac{\theta-\phi}{2}\right)$. (0

2 a. Find the n^{th} derivative of $y = e^{ax}\cos(bx+c)$. (06

b. If $y = e^{m\sin^{-1}x}$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. (07

c. Expand $\log(1+\sin x)$ in powers of x, by using Maclaurin's theorem. (07

3 a. If $z = e^{ax+by}f(ax-by)$, then show that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$. (06

b. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. (0

c. If $u = \tan^{-1}x + \tan^{-1}y$ and $v = \frac{x+y}{1-xy}$ find $\frac{\partial(u,v)}{\partial(x,y)}$

4 a. With usual notation, prove that $\tan\phi = r\frac{d\theta}{dr}$

b. Find the angle between the curves $r = a(1-\cos\theta)$ and $r = 2a\cos\theta$.

c. Find the pedal equation of the curve $r = a(1+\cos\theta)$.

5 a. Obtain the reduction formula for $\int \sin^n x\,dx$, where n is a positive integer.

b. Evaluate $\int_0^1 \frac{x^9}{\sqrt{1-x^2}}dx$.

c. Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}dz\,dy\,dx$.

6 a. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

b. Show that $\int_0^{\pi/2} \sqrt{\sin\theta} \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}}\, d\theta = \pi$.

c. Evaluate $\int_0^{\infty} \frac{dx}{1+x^4}$ in terms of Beta functions.

7 a. Solve $\frac{dy}{dx} = \sin(x+y)$.

b. Solve $x dy - y dx = \sqrt{x^2+y^2}\, dx$.

c. Solve $(x^2 - 4xy - 2y^2)\, dx + (y^2 - 4xy - 2x^2)\, dy = 0$.

8 a. Solve $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$.

b. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \cos 2x$.

c. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \cos x$.

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